

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 12**

**JUNE 2022**

**MATHEMATICS P2**

**MARKS: 150**

**TIME: 3 hours**



This question paper consists of 12 pages and an answer book of 19 pages.

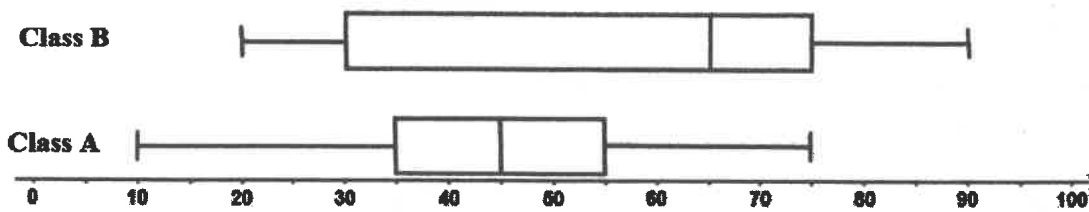
**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.
3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
4. Answers only will not necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. Number the answers correctly according to the numbering system used in this question paper.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

**QUESTION 1**

The box and whisker diagrams below show the Mathematics results of class A and class B in the June Examination. It is also given that class B has a Median of 65%.



- 1.1 Which class had the top learners? (1)
  - 1.2 Determine which class had the greatest Inter Quartile Range (IQR). (1)
  - 1.3 What percentage of class A scored less than 60%? (1)
  - 1.4 If all the learners in class A were given an extra 5%, what would happen to the standard deviation of the marks in class A? (1)
  - 1.5 Determine the semi-interquartile range of class B. (1)
- [5]**

**QUESTION 2**

A group of 30 pupils was asked to complete an obstacle course at their Grade 11 camp. The times (in seconds) taken by the pupils to complete the obstacle course are given in the table below.

Time taken	$60 \leq t < 90$	$90 \leq t < 120$	$120 \leq t < 150$	$150 \leq t < 180$	$180 \leq t < 210$
No. of pupils	3	6	7	8	6

- 2.1 Complete the cumulative frequency table for above data in the SPECIAL ANSWER BOOK. (1)
  - 2.2 Draw a cumulative frequency curve for the above data on the grid provided. (4)
  - 2.3 Indicate on your graph where you would read off:
    - 2.3.1 The number of pupils that took 135 seconds to complete the course (Use the letter A) (1)
    - 2.3.2 The value of  $t$  if 60% of the pupils took less than  $t$  seconds to complete the obstacle course. (Use the letter B) (1)
    - 2.3.3 The 75th percentile. (Use the letter C) (1)
- [8]**

**QUESTION 3**

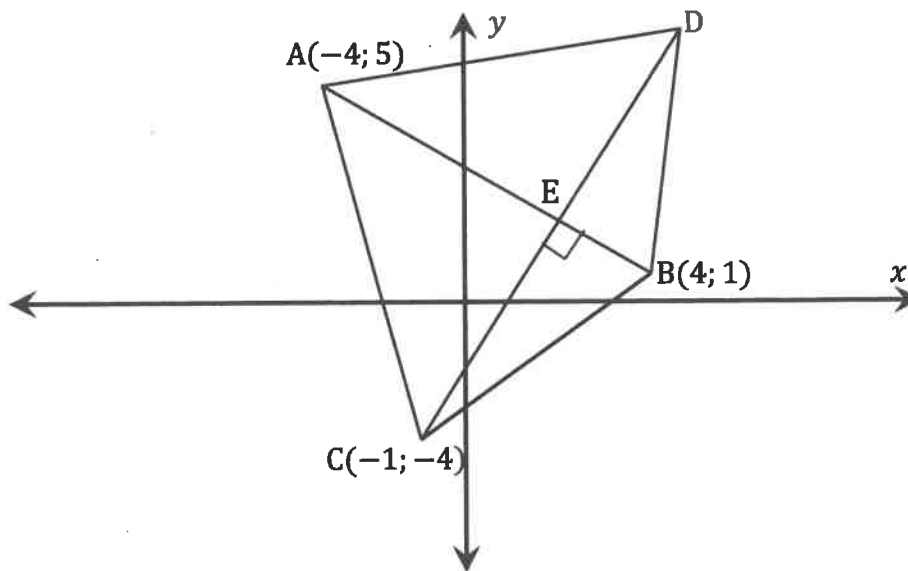
Consider the following set of four positive whole numbers and their frequency.

Scores	$x + 3$	$2x$	$x - 1$	6
Frequency	4	3	2	2

- 3.1 Determine the median score. (1)
- 3.2 Determine the mean in terms of  $x$ . (3)
- 3.3 If only the scores are taken into consideration (without frequency), determine the standard deviation if it is given that  $x = 5$ . (2)
- [6]

**QUESTION 4**

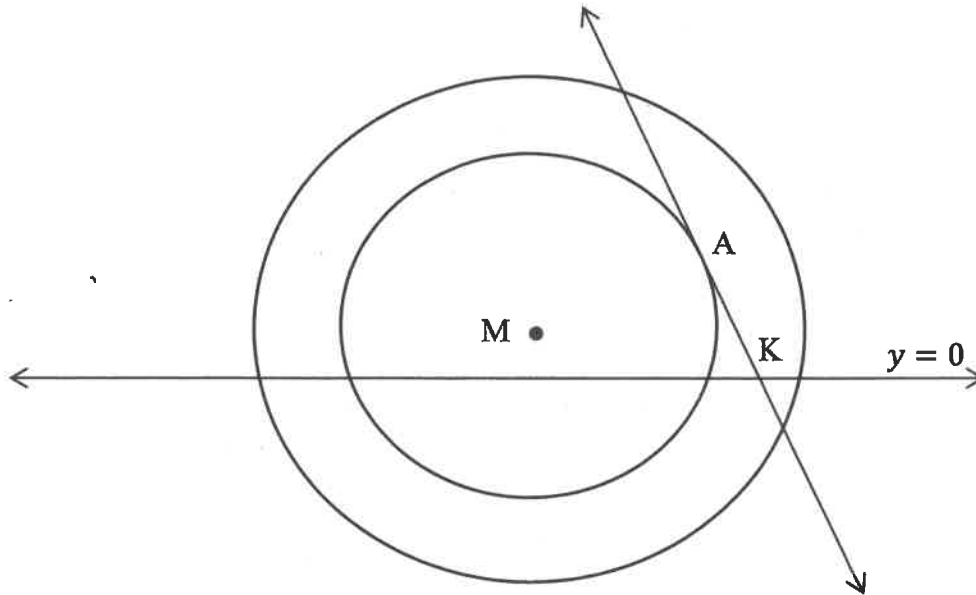
In the diagram below, the coordinates of  $A(-4; 5)$ ,  $C(-1; -4)$  and  $B(4; 1)$  are the vertices of a triangle in a Cartesian plane.  $CE \perp AB$  with  $E$  on  $AB$ .  $E$  is the midpoint of straight-line  $CD$ .



- 4.1 Determine the gradient of  $AB$ . (2)
- 4.2 Determine the equation of  $CD$ . (4)
- 4.3 Determine the coordinates of  $E$ . (6)
- 4.4 Determine the coordinates of  $D$ . (2)
- 4.5 Determine the equation of the straight line passing through point  $D$  and parallel to  $AC$ . (4)
- 4.6 Determine, by showing ALL calculations, whether the  $x$ -intercept of the straight line  $CD$  also lies on the altitude (perpendicular height) from  $A$  to  $BC$ . (6)
- [24]

**QUESTION 5**

In the figure below, M is the common centre of two circles. The larger circle has equation  $x^2 + y^2 = 4y - 2x + 44$ . The smaller circle touches the straight line  $y = -x + 5$  at point A. The straight line  $y = 0$  cuts both circles.



- 5.1 Determine the coordinates of M. (4)
- 5.2 Determine the coordinates of A. (5)
- 5.3 Determine the equation of the smaller circle. (3)
- 5.4 Write down the coordinates of K. (1)
- 5.5 The straight line  $y = -x + 5$  meets the straight line  $y = 0$  at point K. Determine the area of  $\Delta AMK$ . (3)

[16]

**QUESTION 6**

6.1 If  $\cos 26^\circ = \frac{1}{p}$ . Determine the following in terms of  $p$ .

6.1.1  $\sin 26^\circ$  (3)

6.1.2  $\cos 52^\circ$  (3)

6.1.3  $\tan^2 64^\circ \times (p + 1)$  (4)

6.2 Simplify:  $\frac{\sin(-\beta) + \sin(360^\circ - \beta)}{\sin(180^\circ - \beta) + \sin 180^\circ}$  (5)

6.3 Determine the value of  $p$ , correct to two decimal places if  $\theta = 82^\circ$  and  $2p \tan\left(\frac{\theta}{2}\right) = \sin(2\theta)$  (3)

6.4 Prove the identity:  $4 \sin \theta \cdot \cos^3 \theta - 4 \cos \theta \cdot \sin^3 \theta = \sin 4\theta$  (6)  
[24]

**QUESTION 7**

Given:  $f(x) = \sin(x - 30^\circ)$  and  $g(x) = \cos 3x$ .

7.1 Solve for  $x$ :  $\cos 3x = \sin(x - 30^\circ)$  for  $x \in [-60^\circ; 180^\circ]$ . (7)

7.2 Draw the graphs of  $f$  and  $g$  for  $x \in [-60^\circ; 180^\circ]$  on the grid provided. (6)

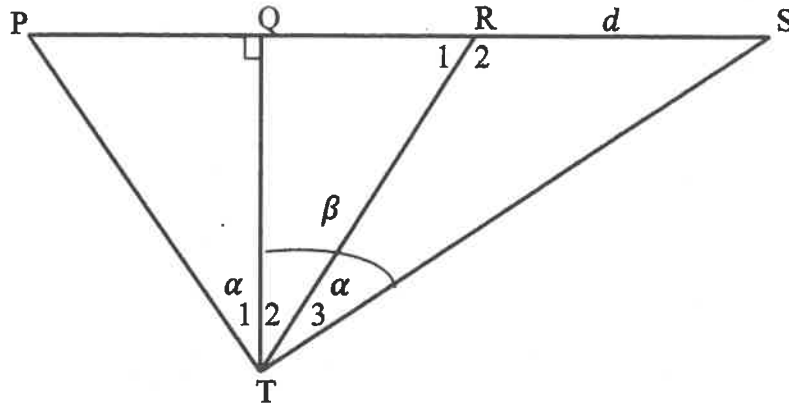
7.3 Use your graph and the answers to QUESTION 7.1 to answer the following question.

For which value(s) of  $x$  is  $f(x) \times g(x) < 0$ ? (4)  
[17]

**QUESTION 8**

Refer to the figure shown below. PQRS forms a straight road with TQ another road that is perpendicular to road PQRS. The distance,  $RS = d$  kilometres.

$\hat{T}_1 = \hat{T}_3 = \alpha$  and  $Q\hat{T}S = \beta$

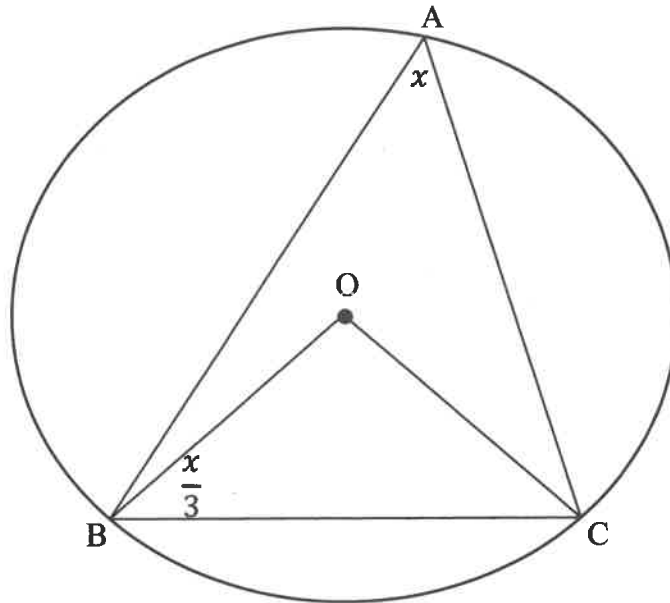


- 8.1 Write down the size of  $Q\hat{T}R$  in terms of  $\alpha$  and  $\beta$ . (1)
  - 8.2 In  $\Delta SQT$ , write down the size of  $\hat{S}$ . (1)
  - 8.3 In  $\Delta PQT$ , write down the size of  $\hat{P}$ . (1)
  - 8.4 Determine the length of  $RT$  in terms of  $\alpha$  and  $\beta$ . (3)
  - 8.5 Hence, or otherwise, show that:  $PR = \frac{d \cos \beta \sin \beta}{\sin \alpha \cdot \cos \alpha}$  (3)
- [9]**

## QUESTION 9

9.1 Complete the statement: The angle at the ... is equal to two times the angle at the circumference of the circle. (1)

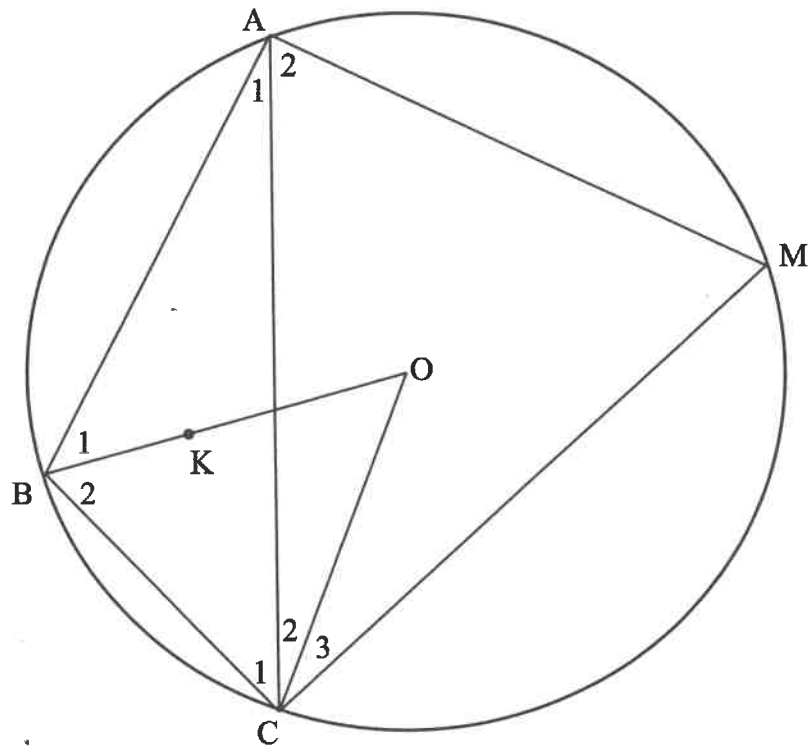
9.2 See diagram below. O is the centre of the circle with points A, B and C on the circumference of the circle.  $\widehat{BAC} = x$  and  $\widehat{OBC} = \frac{x}{3}$ . Determine, with reasons, the value of  $x$ .



(6)



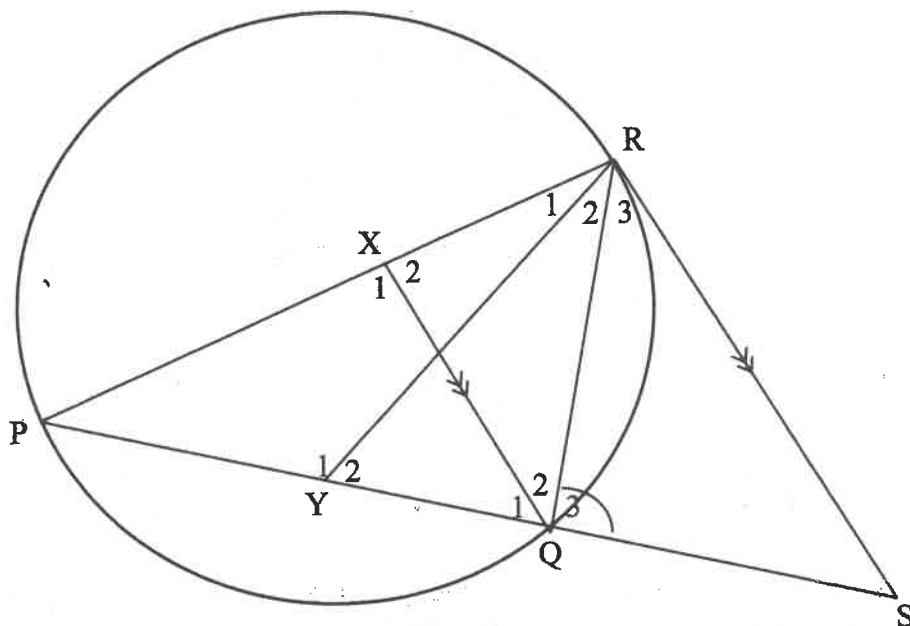
- 9.3 In the diagram below, O is the centre of the circle passing through A, B, C and M. K is the centre of the circle (not drawn) passing through points A, B and C of  $\Delta ABC$  such that K lies on radius BO.  $\hat{A}_1 = 30^\circ$ . BO bisects  $\widehat{AC}$ .



- 9.3.1 Determine the size of  $\hat{B}_1$ . (Supply reasons for your answer.) (5)
- 9.3.2 Prove that  $\hat{M} = 2\hat{A}_1$  (3)
- [15]

## QUESTION 10

In the diagram below, P, Q and R are points on a circle. YR bisects  $\widehat{PRQ}$  with Y on PQ. PQ is produced to meet RS at S such that  $SR = SY$ .  $QX \parallel SR$ .



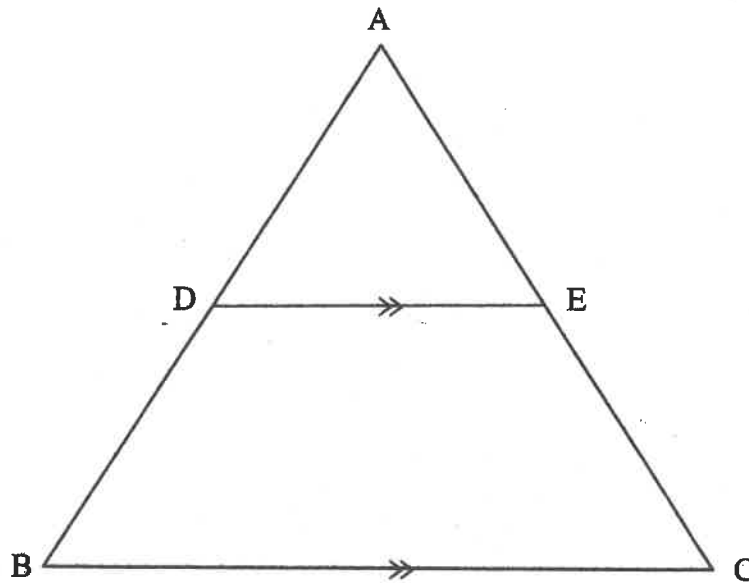
10.1 Prove that  $SR$  is a tangent to the circle at R. (6)

10.2 Prove that  $QR$  is a tangent to the circle passing through Q, X and P. (3)

[9]

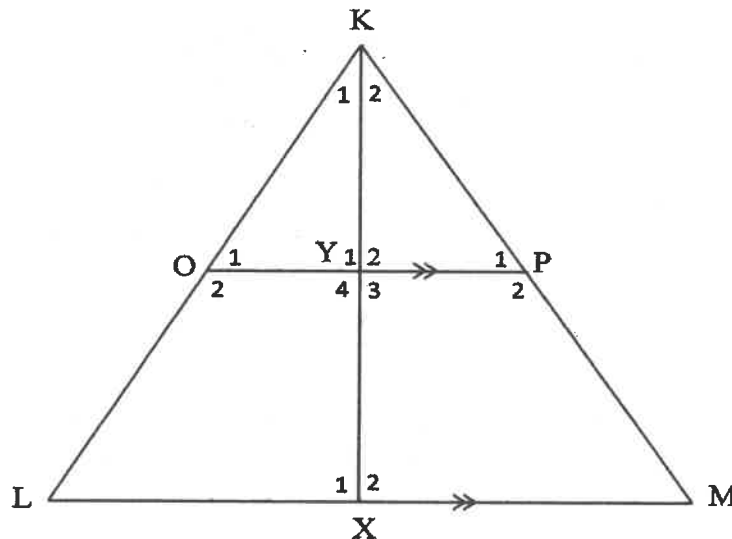
**QUESTION 11**

- 11.1 In the diagram below D and E are points on sides AB and AC of  $\Delta ABC$  such that  $DE \parallel BC$ . Use the diagram to prove the theorem which states that  $\frac{AD}{DB} = \frac{AE}{EC}$ .



(6)

- 11.2 In the diagram below,  $OP \parallel LM$  such that the area of  $\Delta KOP =$  area of quadrilateral  $OLMP$ .  $KYX$  is perpendicular to  $OP$  and  $LM$  at  $Y$  and  $X$  respectively.



Prove that:

11.2.1  $\Delta KOP \parallel \Delta KLM$  (3)

11.2.2  $\frac{KY}{KX} = \frac{OP}{LM}$  (2)

11.2.3  $\frac{KO}{KL} = \frac{1}{\sqrt{2}}$  (6)

[17]

**TOTAL: 150**

## INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$F = \frac{x \left[ (1+i)^n - 1 \right]}{i}$$

$$P = \frac{x \left[ 1 - (1+i)^{-n} \right]}{i}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n} \quad \sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$